

range, the ratio seems to be almost constant. This implies therefore that there exists to a good approximation a reduced θ_D curve of the form

$$\theta_D = F(V)f(T/\theta_D). \quad (13)$$

Since $-\partial \ln \theta / \partial \ln V = 2.4$ and is very nearly constant over the whole volume range investigated, we can write $F(V) \propto V^{-2.4}$. These results mean that a Gruneisen equation of state describes the data quite well over a wide range of volumes and temperatures.

As already mentioned, it should be possible to calculate values of θ_0 from the elastic constants of the solid at the density of interest. Although we do not know the complete elastic constants of solid helium we do know its compressibility as a function of volume. From this alone it is possible to calculate an approximate value of θ_0 by making some assumption about how Poisson's ratio varies with volume. The simplest relation of this kind implies that

$$\theta \propto (a/\beta m)^{\frac{1}{2}}, \quad (14)$$

where a is the lattice parameter, β the compressibility, and m the mass of the atom. In table 12 we compare the experimental values of θ_D evaluated at the same reduced temperature ($\theta_D/T = 18$) with the corresponding values of $(a/\beta m)^{\frac{1}{2}}$ for both helium isotopes.

TABLE 12. COMPARISON OF θ_D FROM SPECIFIC HEATS WITH θ ESTIMATED FROM THE COMPRESSIBILITY

The values of θ_D are taken at the same reduced temperature $T/\theta_D = 18$. The values in brackets are extrapolated from slightly higher reduced temperatures.

V (cm ³ /mole)	θ_D (°K)	$\theta_D(\beta m/V^{\frac{1}{2}})^{\frac{1}{2}}$
	³ He	
12.57	101.2	500
13.33	89.3	497
13.56	85.8	496
14.16	78.0	495
14.98	68.1	491
15.72	60.5	491
16.71	(51.9)	482
16.87	(50.9)	483
17.02	(49.6)	480
	⁴ He	
11.77	99.9	499
12.22	91.9	509
14.55	62.0	526
16.25	(46.6)	510

(Instead of a , the lattice parameter, we have written $V^{\frac{1}{2}}$.) The last column of the table gives the product $\theta_D(\beta m/V^{\frac{1}{2}})^{\frac{1}{2}}$. Although for ³He it shows small systematic changes and for ⁴He some bigger, apparently random, changes, this product remains rather constant over the whole range. We may therefore conclude that the volume dependence of θ reflects quite closely the volume dependence of β . By putting in a reasonable value for the constant of proportionality in equation (14) (cf., for example, Mitra & Joshi 1961) it is found that the magnitude of θ so calculated is similar to the values of θ_0 indicated by the extrapolations in figures 7 and 8. Since the theory is only approximate this agreement is not very significant but it does perhaps suggest that if there is a low temperature anomalous contribution to the specific heat as found by Heltemes & Swenson it is *not* due to the lattice vibrations.